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BINOMIAL THEOREM

1. STATEMENT OF BINOMIAL THEOREM

$$(x + a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + \dots + {}^nC_n a^n \quad (\text{where } n \in \mathbb{N})$$

$$\cdot {}^nC_0, {}^nC_1, {}^nC_2, \dots, {}^nC_n \text{ are binomial coefficients } {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$\text{General Term} = T_{r+1} = {}^nC_r x^{n-r} a^r$$

- There are $(n+1)$ terms in the expansion of $(x + a)^n$.
- The sum of powers of a and x in each term of expansion is n .
- The binomial coefficients in the expansion of $(x + a)^n$ equidistant from the beginning and the end are equal.

2. GREATEST BINOMIAL COEFFICIENT

- If n is even : When $r = \frac{n}{2}$ i.e. ${}^nC_{n/2}$ takes maximum value.
- If n is odd : $r = \frac{n-1}{2}$ or $\frac{n+1}{2}$ i.e. ${}^nC_{\frac{n-1}{2}} = {}^nC_{\frac{n+1}{2}}$ and take maximum value.

3. MIDDLE TERM OF THE EXPANSION

- If n is even $T_{\left(\frac{n}{2}+1\right)}$ is the middle term. So the middle term $T_{\left(\frac{n}{2}+1\right)} = {}^nC_{n/2} x^{n/2} y^{n/2}$
- If n is odd $T_{\left(\frac{n+1}{2}\right)}$ and $T_{\left(\frac{n+3}{2}\right)}$ are middle terms. So the middle terms are

$$T_{\left(\frac{n+1}{2}\right)} = {}^nC_{\left(\frac{n-1}{2}\right)} x^{\frac{n+1}{2}} y^{\frac{n-1}{2}} \text{ and } T_{\left(\frac{n+3}{2}\right)} = {}^nC_{\left(\frac{n+1}{2}\right)} x^{\frac{n-1}{2}} y^{\frac{n+1}{2}}$$

4. TO DETERMINE A PARTICULAR TERM IN THE EXPANSION

In the expansion of $\left(x \pm \frac{1}{x}\right)^n$, if x^m occurs in T_{r+1} , then r is given by

$$r = \frac{n - m}{+}$$

The term which is independent of x , occurs in T_{r+1} , then r is $r = \frac{n}{+}$

5. BINOMIAL COEFFICIENT PROPERTIES

- (1) $C_0 + C_1 + C_2 + \dots + C_n = 2^n$
- (2) $C_0 - C_1 + C_2 - C_3 + \dots + (-1)^n C_n = 0$

$$(3) C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$$

$$(4) C_0 C_r + C_1 C_{r+1} + \dots + C_{n-r} C_n = {}^{2n}C_{n-r} = \frac{2n!}{(n+r)!(n-r)!}$$

$$(\text{if } r = 0) \quad C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \frac{2n!}{n!n!}$$

$$(\text{if } r = 1) \quad C_0 C_1 + C_1 C_2 + C_2 C_3 + \dots + C_{n-1} C_n = {}^{2n}C_{n-1} = \frac{2n!}{(n+1)!(n-1)!}$$

$$(5) C_1 + 2C_2 + 3C_3 + \dots + nC_n = n \cdot 2^{n-1}$$

$$(6) C_1 - 2C_2 + 3C_3 - \dots + (-1)^n \cdot nC_n = 0$$

$$(7) C_0 + 2C_1 + 3C_2 + \dots + (n+1)C_n = 2^{n-1} (n+2)$$

$$(8) C_0 + \frac{C_1}{2}x + \frac{C_2}{3}x^2 + \frac{C_3}{4}x^3 + \dots + \frac{C_n}{n+1}x^n = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

$$\Rightarrow C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1} \quad (x = 1)$$

$$\Rightarrow C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \dots + \frac{(-1)^n \cdot C_n}{n+1} = \frac{1}{(n+1)} \quad (x = -1)$$

$$(9) C_0^2 - C_1^2 + C_2^2 - C_3^2 + \dots + (-1)^n Cx^2 = \begin{cases} 0 & \text{if } n \text{ is odd} \\ (-1)^{n/2} n C_{n/2} & \text{if } n \text{ is even} \end{cases}$$

6. NUMERICALLY GREATEST TERM OF BINOMIAL EXPANSION

$$(a+x)^n = C_0 a^n + C_1 a^{n-1} x + \dots + C_n x^n.$$

$$\left| \frac{T_{r+1}}{T_r} \right| = \left| \frac{{}^n C_r}{{}^n C_{r-1}} \right| \left| \frac{x}{a} \right| = \left| \frac{n-r+1}{r} \right| \left| \frac{x}{a} \right|$$

$$\text{Take } \left| \frac{n-r+1}{r} \right| \left| \frac{x}{a} \right| \geq 1 \quad \left\{ \text{As } |T_{r+1}| \geq |T_n| \right\}$$

$$r \leq \frac{n+1}{1 + \left| \frac{a}{x} \right|}$$

$$\text{So greatest term will be } T_{r+1} \text{ where } r = \left[\frac{n+1}{1 + \left| \frac{a}{x} \right|} \right]$$

[.] denotes greatest integer function.

Note : If $\frac{n+1}{1 + \left| \frac{a}{x} \right|}$ itself is a natural number, then $T_r = T_{r+1}$ and both the terms are numerically greatest.

7. BINOMIAL THEOREM FOR ANY INDEX

If $n \in \mathbb{Q}$, $|x| < 1$, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r + \dots \infty$$

Note : In this case there are infinite terms in the expansion.

Some Important Expansions :

If $|x| < 1$ and $n \in \mathbb{Q}$ then

$$(a) (1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \dots + \frac{n(n+1)\dots(n+r-1)}{r!}x^r + \dots$$

$$(b) (1+x)^{-n} = 1 - nx + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \dots + \frac{n(n+1)\dots(n+r-1)}{r!}(-x)^r + \dots$$

By putting $n = 1, 2, 3$ in the above results, we get the following results-

$$\cdot (1-x)^{-1} = 1 + x + x^2 + x^3 + \dots + x^r + \dots$$

$$\cdot (1+x)^{-1} = 1 - x + x^2 - x^3 + \dots + (-x)^r + \dots$$

$$\cdot (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots + (r+1)x^r + \dots$$

$$\cdot (1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots + (r+1)(-x)^r + \dots$$

$$\cdot (1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \dots + \frac{(r+1)(r+2)}{2!}x^r + \dots$$

$$\cdot (1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \dots + \frac{(r+1)(r+2)}{2!}(-x)^r + \dots$$

8. SOME IMPORTANT RESULTS

(i) If the coefficient of the r^{th} , $(r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ terms in the expansion of $(1+x)^n$ are in H.P. then

$$n + (n-2r)^2 = 0$$

(ii) If coefficient of r^{th} , $(r+1)^{\text{th}}$, and $(r+2)^{\text{th}}$ terms in the expansion of $(1+x)^n$ are in A.P. then

$$n^2 - n(4r+1) + 4r^2 - 2 = 0$$

(iii) Number of terms in the expansion of $(x_1 + x_2 + \dots + x_r)^n$ is ${}^{(n+r-1)}C_{r-1}$